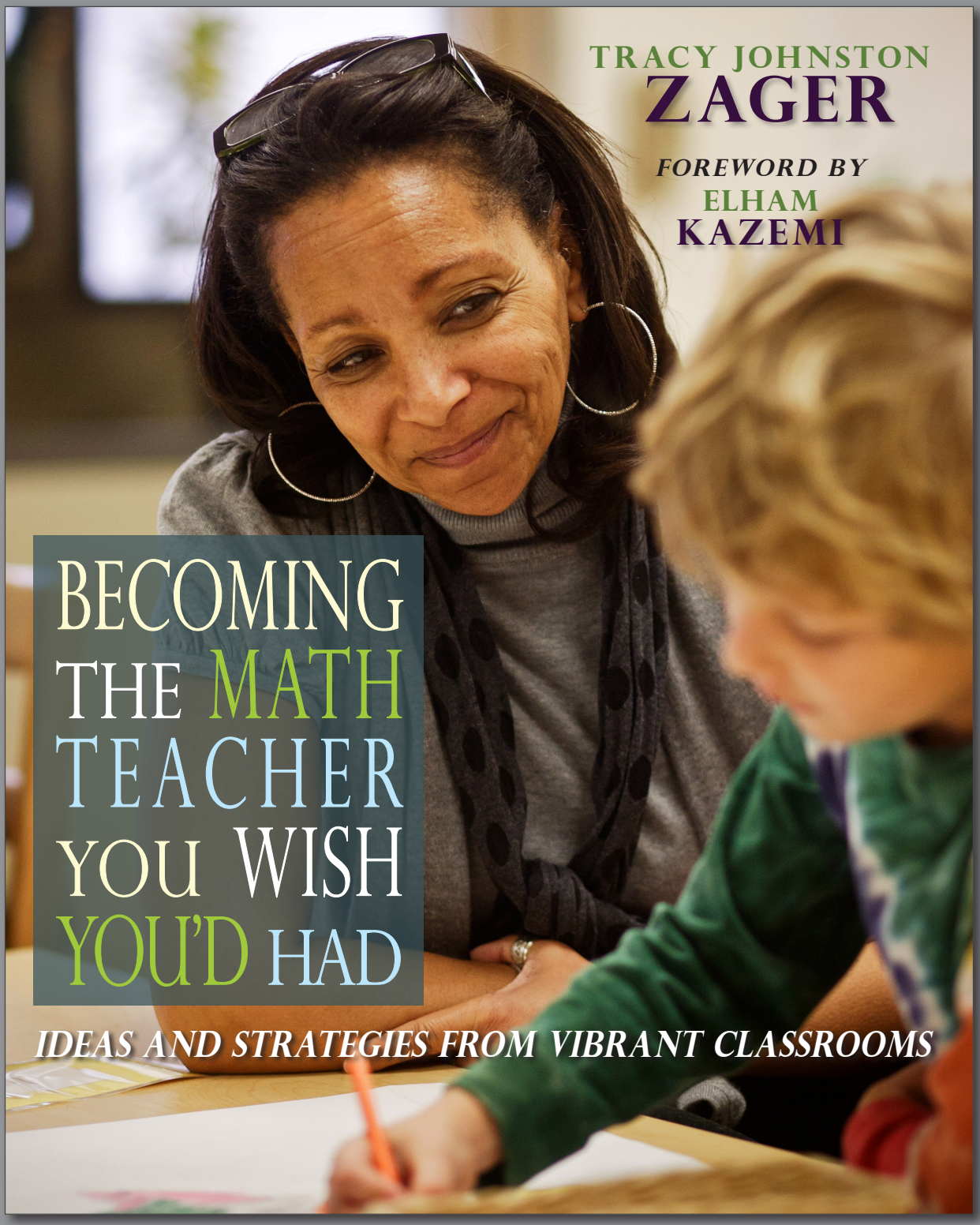
Study Guide



Professional development training for teachers provided by Stenhouse Publishers

Stenhouse Publishers
www.stenhouse.com

Contents

Introduction

When I wrote *Becoming the Math Teacher You Wish You'd Had,* I decided to create a meaty resource for math teachers, coaches, professors, and specialists. There's a lot of content, and I'm hoping each professional learning community, book study group, and individual reader will make the book work for them by focusing on the parts that meet their needs best. That's why I'm happy with the decision to have a grand scale and scope: I packed a lot in the book knowing you'd have plenty of ideas to choose from.

I ran a risk, however, which is that the length and substance might feel overwhelming. How can you get your arms around a book like this?

In this study guide, I will offer some suggestions for how you might approach reading this book in a professional learning community, book study, online chat, or course. I've identified fruitful places to do the following:

- Pause and discuss or write. I've given you guiding questions.
- Facilitate activities with your colleagues.
- Try an idea with your students and report back to one another and/or to me and other readers online.

I'm hoping these suggestions will be helpful, but please remember, these are just suggestions. Feel free to pick and choose what you like, or come up with different ideas entirely. I'd be thrilled to hear about how you choose to read it and make it your own. We need to do this work together.

To share your learning and wondering with me, look for me on Twitter at @tracyzager or by using the hashtag #becomingmath. I have posts and additional resources organized by chapter at my blog, tjzager.com, where I invite you to connect with other readers and with me. I'll moderate discussions, read comments, and respond to ideas and questions. I'll provide a place for you to talk about the results of any ideas you try. I'll also post information about online discussion groups and forums as I hear about them, and I plan to drop in whenever I can. I'm grateful that modern technology makes it possible for us to collaborate across space and time, and I look forward to learning together with you wherever and whenever we might meet.

Thinking About the Table of Contents

As I said in the book, I think of its structure in terms of months and seasons. The thirteen chapters are the months that comprise five larger seasons. For example, taking risks, making mistakes, and precision make up one season. Intuition, reasoning, and proof are another season.

One way to handle the book's length is to read a season at a time, pausing between seasons to try strategies and connect ideas.

Chapter 1: Breaking the Cycle
Chapter 2: What Do Mathematicians Do?

Chapter 3: Mathematicians Take Risks
Chapter 4: Mathematicians Make Mistakes
Chapter 5: Mathematicians Are Precise

Chapter 6: Mathematicians Rise to a Challenge
Chapter 7: Mathematicians Ask Questions
Chapter 8: Mathematicians Connect Ideas

Introduction (continued)

- Chapter 9: Mathematicians Use Intuition
- Chapter 10: Mathematicians Reason
- Chapter 11: Mathematicians Prove

- Chapter 12: Mathematicians Work Together and Alone
- Chapter 13: "Favorable Conditions" for All Math Students

Also, while the book definitely builds in sophistication from start to finish and ideas grow atop one another, each chapter can stand alone. Feel free to jump around to meet your needs.

Chapter 1: Breaking the Cycle

In Chapter 1, I talked about the relationship between how we were taught math and how we teach math. In your group, it's essential you create a safe climate built around trust and honesty in order to discuss these issues. A small percentage of teachers feel fine about math or enjoyed it as students, but most teachers have had at least a mix of negative and positive experiences, and they may still feel shame, hurt, or embarrassment. You can help them quite a bit by giving them a supportive venue in which they can address those feelings and move on productively.

I've suggested two different starting activities below. Each encourages teachers to recall their experiences. We bring these stories into classrooms with us, whether or not we realize it.

I would recommend that teachers do either the math autobiography or the word clouds, but not both because they might feel redundant.

Discussion Questions

Page 5 It's worth focusing everyone's attention on the mathematicians' word cloud. Any surprises? Give colleagues a chance to discuss or write about their reactions. Is there a word that speaks to them? They might want to choose one word that doesn't currently feel like it's part of their math classroom but should be. Think about how to incorporate that one word this year.

Page 8 Ask colleagues to take note of their feelings about the three approaches in the book (study of mathematicians, learning from other teachers, engaging with mathematics). Which approach are they most comfortable with? Excited about? Wary of? Why? Ask what would help them open themselves to all three.

Activities

Page 5 **Math Autobiographies**

Some teachers (and students!) have found it helpful to write a math autobiography. The basic structure is that each person writes his or her personal story with mathematics. You can ask participants to start with their earliest memories of math and work toward the present day. Their writing does not need to be exhaustive but should include the memories that made a big impact on them. I've posted a few examples at tjzager.com and stenhouse.com/becomingmathteacher.

Page 5 **Word Clouds**

Take a long look at the word clouds in Figures 1.1 and 1.2. Discuss or write about them as feels natural. In the book, I said, "Admittedly, these images describe the extremes of a continuum, and I expect most of our personal mental maps about mathematics fall somewhere in between." It can be helpful for teachers to create their own word clouds to examine their associations. Give every participant a blank piece of paper with the word "math" printed in the center (make your own or use the one provided at tjzager.com), and ask them to jot down words or images or passages of text in response.

These word clouds and autobiographies are highly personal, so don't expect people to share, but certainly welcome anyone who wants to talk.

Chapter 1: Breaking the Cycle (continued)

Additional Resources

At stenhouse.com/becomingmathteacher and at tjzager.com, you'll find a collection of supplemental resources that may come in handy for further thinking and discussion. I keep the links fresh, so the contents will change, but you will certainly find:

- A link to a powerful poem from Jane Kenyon: "Trouble with Math in a One-Room Country School"
- A video of a short talk I gave about the ideas in this chapter
- Blog posts and videos about people's experiences in math that may spark productive discussion

CHAPTER 2: What Do Mathematicians Do?

In Chapter 2, I shared informal action research my colleague Debbie Nichols and I conducted with her primary students. As a coach, I shared this work and engaged in these activities with the larger staff. I found that introducing Deb's unit as something practical teachers could try with their students to launch math class or to create positive norms for math class gave me just enough buy-in that teachers were willing to give it a go. It's important we honor how busy teachers are, and how important it is to engage in activities that feel relevant to their work. At the same time, it's essential we open (and keep open) the larger discussions around our identities as mathematicians, our understanding of the discipline of mathematics, and our framing of mathematics for our students. This mini-unit was definitely tipped toward the larger discussions, but it was just practical enough that they were willing to try.

Therefore, the staff did all the activities described in this chapter. They read the picture books. They played with the mathematical Internet. They reflected on what they were learning about what mathematics is and what mathematicians do. Most staff realized that this time spent on the larger context was time well invested, because it gave them a framework for all the obviously practical, more sophisticated teaching strategies to come.

Discussion Questions

Page 9 Before reading the chapter, discuss how you would answer the question, *What does it mean to be good at math?* How might your students answer? Written reflection might be productive as well.

Page 10 What have you observed about how students use the phrase "This is easy" in your classrooms? What has been the effect?

Page 12 What have you noticed about language—either from teachers or peers—that deflates the positive climate you want to create? Have you noticed similar trends to what Deb and I saw? What do you think about students' responses when we opened the conversation?

Page 23 Discuss or write about Atticus's response to the Fruit by the Foot play. What was he afraid of? Why? What can we do to alleviate that fear?

Page 28 Discuss or write about the last section of the chapter, in which I talk about the connection between the unit and the rest of the year. We commonly post inspirational posters or say words that sound encouraging but send very different messages with our actions. We need to become aware of and reflect on this tendency so we can change it.

You might spark a lively discussion by sharing an example of this dissonance: an "inspirational" poster about curiosity alongside a mind-numbing worksheet would do nicely. This is a provocative conversation to facilitate, and it's essential your tone is reflective and self-aware rather than judgmental or critical. If you can share an example from your own practice, all the better. You want to position yourself as a learner among your colleagues, one who is also working to bring your actions and your messages into alignment.

Activities

Page 18 **Children's Books Jigsaw**

Get your hands on as many copies of these books as you can. Since Deb and I did this work, I've learned of a strong fifth book, Kathryn Lasky's *The Librarian Who Measured the Earth.* Feel free to add that one in as well.

CHAPTER 2: What Do Mathematicians Do? (continued)

Set up a jigsaw structure with your colleagues. Take a few moments to introduce each book to everyone, and then ask them to choose the most appealing book. They should read it with a colleague and talk. When they've finished, have them reshuffle into groups so every book is represented. Ask participants to take a few minutes to talk about what stood out to them during the reading.

Throughout, focus people's attention on the essential questions:

What do mathematicians do?
What does it mean to be good at math?
What is mathematics?

It's helpful to emphasize the *verbs* in the books. What do mathematicians do? Come together and discuss or write about the verbs Debbie's children used (Figure 2.1, page 20 in the text) as well as the verbs that struck teachers.

Page 25 **Internet Wealth**

Take a good chunk of time together to investigate the online resources that appeal most, such as Math Munch, Numberphile, Vi Hart, and the Math Photo challenge (tjzager.com or stenhouse.com/becomingmathteacher). You might each share a quick (two to five minutes each) favorite find. It's helpful to project the site, not just talk about it.

Page 27 **Scavenger Hunt**

Ask participants to choose an everyday object that they see mathematically and bring it to the next session or share photographs of it through social media. Engage in a gallery walk and open-ended discussion about what people notice and wonder. You might want to host a Math Photo challenge for your school or district (see the Chapter 2 resources at tjzager.com or stenhouse.com/becomingmathteacher).

Calls to Action

Page 13 **Taking on Language**

I invite you to open a similar conversation with your students around words such as *easy, hard, fast, slow, right, wrong*, or around the question, *What does it mean to be good at math?* If you can, record the audio or video, and then transcribe or summarize it. What did you learn? Share with your colleagues and/or online in the "Becoming the Math Teacher You Wish You'd Had" chapter discussions at tjzager.com (Chapter 2). You can comment directly or share a link to your blog post so other readers can learn from your experience.

Additional Resources

At stenhouse.com/becomingmathteacher and at tjzager.com, you'll find a collection of supplemental resources that may come in handy for further thinking and discussion. I keep the links fresh, so the contents will change, but you will certainly find:

- A video of Maya and Daphne (my kids) discussing what they found on their scavenger hunt
- All the websites mentioned in this chapter
- A few articles about babies and math, if the mentions of those studies piqued your interest

CHAPTER 3: Mathematicians Take Risks

We're about to get a lot more specific. My hope is that the opening chapters started conversations and set the tone. Now we're ready to focus on one habit of mind at a time. We'll begin by sitting in on classes as teachers engage students with more obviously mathematical content. We'll analyze specific teacher moves and actions that create conditions for powerful mathematics learning. And we'll look at the results through student dialogue and work.

I ramped up the content gently on purpose, knowing some portion of the readership may feel nervous. My first priority is not to shut those readers down. If some of your colleagues have been chomping at the bit to get to meatier examples, I hope that, by the end of this chapter, they'll have felt challenged and provoked. The sophistication will continue to rise from here on out.

Discussion Questions

Page 30 Discuss or write about the paragraph that starts, "On the inside, I was angry." What do you make of my speculation that Mr. Duncan felt his authority and expertise were under attack? Have you been in Mr. Duncan's shoes? How did you handle it?

Page 31 Discuss or write about obedience versus risk taking in mathematics. What came to mind when you read this passage? What are you thinking now? What new questions do you have?

Page 32 What do you make of my assertion that it's immaterial if the mathematics that students are grappling with has been settled by other mathematicians? Discuss or write about the statement, "Whether or not other mathematicians have had the same idea before is *completely irrelevant to that student.*"

Page 32 Have you ever complained that your students won't try? What patterns have you noticed? What strategies have you tried?

Page 45 The downsides of the words *smart* and *easy* keep emerging. Discuss or write about how you might change your usage of these words.

Page 49 How does Shawn leverage his role as a teacher and a leader to encourage risk taking?

Page 49 Teachers who encourage and nurture students are often framed as wishy-washy, loose, or lax. How does Shawn's example influence your thinking about this caricature?

Page 50 Does your curriculum encourage mathematical risk taking or does it preclude it?

Page 50 What do you make of the distinction between mathematical risks and social risks? Does it ring true to you?

Page 51 Discuss or write about the implications of students' unplanned risks. What can you do to gain comfort with students' unexpected risks? How might you plan your lessons so you'll be able to react positively and think on your feet, while still meeting your objectives? For example, if you'd been Melanie Wood's teacher, how might you have reacted supportively?

CHAPTER 3: Mathematicians Take Risks (continued)

Page 53 If you teach multiple subjects, does the culture and your vision of teaching and learning vary with the subject? Perhaps dig into that question with your colleagues a bit.

Activities

Pages 33-36 **Heidi's Transcript**

Analyze the transcript from Heidi's classroom. What teacher moves did Heidi make? What do you think her goals were for those moves?

Pages 39 **Play with 10**

This is a good chance to engage in your first book-related mathematical play together. It's a gentle introduction, by design, but the openness of the task can lead you anywhere you want to go. What do you notice about each other's work?

Pages 41-43 **Cindy's Student Work**

Analyze Cindy's students' work as a group. What do you notice about their mathematical thinking? Be sure to dig into the math itself. What ideas are students working on? Also, what do you notice about Cindy's comments?

Calls to Action

Page 44 **Feedback Language**

Choose or adapt language from Cindy's comments to try in your teaching. You might want to rehearse this language with your colleagues or on your own so it becomes more natural. How will you incorporate it in your teaching? Oral feedback? Written comments? Once you try it, report back to your colleagues. What did you notice? How did it impact you? Your students? Tell me what you try at tjzager.com.

Pages 51-53 **Make It Safe**

Choose an item or two out of the table. Pick ones that resonated with you. Can you set yourself a goal, try it in your teaching, and reflect on it? Can you come back together with your colleagues and discuss or write about what you learned? I invite you to tell me about it at tjzager.com (Chapter 3).

Additional Resources

At stenhouse.com/becomingmathteacher and at tjzager.com, you'll find a collection of supplemental resources that may come in handy for further thinking and discussion. I keep the links fresh, so the contents will change, but you will certainly find:

- A useful, shareable summary of Carol Dweck's research
- Shawn Towle's quote collection
- A link to Herbert Kohl's powerful essay "I Won't Learn from You"

CHAPTER 4: Mathematicians Make Mistakes

Over the past several years, there has been quite a bit of encouraging talk in the math world about seeing mistakes as a normal part of learning and doing mathematics. Jo Boaler, in particular, has led the charge. Most teachers want to do a better job of handling mistakes in the math classroom but are unsure about how to proceed. In this chapter and the next, I offer several specific teaching techniques to try, as well as a larger argument about why you'd want to.

Discussion Questions

Page 56 Discuss or write about Yau's quote:

> ***Somehow he conveyed the philosophy that making mistakes was normal and that passing from mistake to mistake to truth was the doing of mathematics. And somehow he also conveyed the understanding that once one began doing mathematics it would naturally flow on and on. Doing mathematics would become like a stream.***
>
> What thoughts do these images spark in you and in your colleagues?

Page 57 Discuss or write about the list of teacher's comments. Reflect on the language you currently use when students make mistakes. What language might you use going forward? How will you remind yourself?

Page 63 Consider the paragraph about keeping your face, body language, voice, and words neutral. How do kids pick up cues from you? How can you stay encouraging, honest, and neutral all at the same time?

Page 69 Reflect on the moment Heidi turned to Ayoka and asked, "What do you think?" Did it surprise you? If so, why? What is a more typical response here? How did Heidi's response work out? What message is Heidi conveying to her class about making sense of mathematics? About who holds authority in math class?

Page 71 Discuss or write about Heidi's rationale for digging into Ayoka's work. What do you think about her thought process?

Pages 72–73 Heidi said, "I don't understand it yet." Would you feel comfortable and relaxed saying this to students? To colleagues?

Pages 76–77 Read and reflect on the section "Responsive Planning—Students' Mistakes as Teachers' Guides." How do you determine when a mistake is worthwhile to take up as a class versus handle individually?

Activities

Page 78 **Intuition and Sexism**

Watch Reshma Saujani's *TED Talk* "Teaching Girls Bravery, Not Perfection" together. It's linked on the companion website and blog. Discuss it alongside the closing section of the chapter about equity. How might these ideas influence your teaching?

CHAPTER 4: Mathematicians Make Mistakes (continued)

Calls to Action

Page 57 **Mistakes Versus Errors**

What do you make of the distinction between mistakes and errors? Next time you're teaching, jot down mistakes you hear that reveal conceptual misunderstanding and errors that reveal students' need to work on precision. The line between the two is blurry, but how does distinguishing the two affect the way you think while teaching? Discuss or write about your notes with your colleagues and post them at tjzager.com (Chapter 4).

Page 63 **Opening Discussions**

Consider taking up Julie's approach to opening questions. Next time you start a discussion, ask a thought-oriented question rather than an answer-focused one. What happened after? Share your results at tjzager.com (Chapter 4).

Pages 76–77 **Anticipating Mistakes**

Choose an upcoming lesson. Are there mistakes you can anticipate and welcome? Teaching opportunities you can count on? Gather with some colleagues to plan the lesson, and dedicate some time to anticipating mistakes and opportunities together. Teach the lesson, and then come back together to talk about whether this anticipation helped you and in what way. Report back at tjzager.com (Chapter 4).

Additional Resources

At stenhouse.com/becomingmathteacher and at tjzager.com, you'll find a collection of supplemental resources that may come in handy for further thinking and discussion. I keep the links fresh, so the contents will change, but you will certainly find:

- A link to Michael Pershan's *Math Mistakes* blog, which has extensive sets of mistakes you and your colleagues can discuss
- Short videos and links about some of the mathematicians mentioned in this chapter (Henri Poincaré, Fern Hunt, Shiing-shen Chern)

CHAPTER 5: Mathematicians Are Precise

In Chapter 4, we began looking at mistakes that reveal deep conceptual understanding. In this chapter, we'll look at strategies to teach students how to hold themselves to high standards of clean and clear mathematics within a safe, positive climate.

Discussion Questions

Pages 80–81 Consider this list of related but distinct ideas around precision. Which aspects do your students currently have? Which ones do your students need to work on most?

Pages 81–84 Does your curriculum reduce the cognitive demand on students by over-scaffolding worksheets? What ideas does Shawn's chart give you for quick fixes?

Pages 85–87 Discuss Shawn's instruction around vocabulary. How might reading these examples influence your teaching?

Pages 101–104 What do you make of the idea of authorship in math? Discuss or journal about it.

Pages 104–109 Discuss the section on math facts, delving into the suggested resources if possible. What caught your ear? This is a controversial topic. What are you thinking?

Calls to Action

Page 85 **Improving Games**

Cindy's revised pattern game delayed answer-getting (Daro), increased discourse, and ramped up the rigor. Is there a game you play that would benefit from this treatment? If so, take the time to plan it now. Teach it and then discuss or write. Please tell me about your revisions and results at tjzager.com (Chapter 5).

Pages 90–91 **"I think _____ is unreasonable because _____."**

Plan and teach this powerful warm-up. You'll need to do it a few times for students to internalize the routine. After a few sessions, come back together and discuss how it went, and let me know at tjzager.com (Chapter 5).

Pages 91–93 **"Wowzers!"**

Jen Muhammad regularly externalizes the internal voice she wants students to use. How might you try this strategy in your style? Think about it, try it, and write your reflections or link your blog post at tjzager.com (Chapter 5).

Pages 94–95 **Buddy Checks**

Try implementing *Buddy Checks*. Remember, they take a lot of explicit teaching about caching. Write about your experiences at tjzager.com (Chapter 5).

CHAPTER 5: Mathematicians Are Precise (continued)

Pages 96–101 ***My Favorite No* Adaptations**

Experiment with Jen's *My Favorite No* variations, and maybe think of some new ones. I find it challenging to choose a *My Favorite No* thoughtfully while the kids are watching, as Leah does in the video, especially because I'm aware the kids are just sitting and waiting. Jen's adaptations keep what's great about the idea while giving us some think time so we can choose a productive error. I'd love to hear what variation you chose and why, and how it went at tjzager.com (Chapter 5).

Additional Resources

At stenhouse.com/becomingmathteacher and at tjzager.com, you'll find a collection of supplemental resources that may come in handy for further thinking and discussion. I keep the links fresh, so the contents will change, but you will certainly find:

- Videos from Justin Solonynka's and Chris Luzniak's classrooms
- The original *My Favorite No* from the Teaching Channel
- Resources for fact fluency

CHAPTER 6: Mathematicians Rise to a Challenge

In this chapter, I focus on the role of the problems we assign in supporting or undermining students' agency, independence, and perseverance.

Discussion Questions

Page 113 Discuss or journal about the comparison between Jen's language ("Oooh, a challenge, cool!") and the more typical language on page 113. If this is something you need to work on, how will you start?

Pages 129–131 Discuss or write about the section "Productive Struggle, Be Less Helpful, and Special Education." Does this resonate with your experiences?

Pages 132–133 Discuss the critique of "high, medium, and low" language. These terms are common in education. What would it mean to stop using them?

Pages 133–136 Discuss or journal about the final section of the chapter, "Developing the Skills and Content Expertise to Challenge Students." What resonated with you, personally? What goals might you consider, going forward?

Activities

Pages 113–115 **Be Less Helpful**

As a group, watch Dan Meyer's *TED Talk* and then discuss your curriculum. Where does it fall? How does it measure up? Is it too helpful and in need of a makeover? Ask your colleagues to bring tomorrow's lesson to this session so you can get specific. Where, exactly, is the curriculum too helpful? How might you revise it?

Pages 115–117 **Makeover Blogs**

Ask participants to read Fawn's blog post. Point them to the collection of additional makeover blogs at tjzager.com (Chapter 6) or stenhouse.com/becomingmathteacher. Ask each participant to choose one blog to read. Host a jigsaw or discussion after. What do participants notice? What are they wondering?

Page 128 **Flat Soda**

Take a whack at the soda problem. What makeovers would improve it?

Pages 129–131 **Too Many Rules**

"Without an understanding of the mathematical relationships, each new situation demands a new rule" (Behrend 2001, 37). Behrend cited the rules around lining numbers up for computation. Can you think of other examples? Generate a brainstormed list wherein we teach multiple procedural rules instead of mathematical relationships. Going forward, coaches and teachers can prioritize revising instruction for these areas.

CHAPTER 6: Mathematicians Rise to a Challenge (continued)

Calls to Action

Pages 124–128 **Low Threshold, High Ceiling, Open Middle**

Discuss Papert's image of low-threshold, high-ceiling problems. Does it resonate? Choose a problem from an upcoming lesson and talk about how to lower its threshold and raise its ceiling. What changes did you make? Once you're done, think about the same problem in terms of open or closed beginnings, middles, and ends, as Dan Meyer described. Any further changes?

Page 128 **The Makeover Experiment**

Choose one problem to make over, and rewrite it a few different ways with your team. If you can work it out, the ideal situation would be to test three variants: one colleague would teach the problem as written, one colleague would teach one makeover of the problem, and a third colleague would teach a different makeover of the same problem. If you can work out the sub coverage, try to go to one another's classes and observe the results. If not, perhaps video? No matter what, come together and compare teachers' observations and student work. Which problem brought you the best results? What were the costs and benefits of the different makeovers? Please share what you learned at tjzager.com (Chapter 6).

Pages 131–133 **Margin Symbols**

Experiment with margin symbols. What do you notice? Report back to your colleagues and post at tjzager.com (Chapter 6).

Additional Resources

At stenhouse.com/becomingmathteacher and at tjzager.com, you'll find a collection of supplemental resources that may come in handy for further thinking and discussion. I keep the links fresh, so the contents will change, but you will certainly find:

- Dan Meyer's *TED Talk*, Phil Daro discussing "answer getting," and a BBC documentary from Andrew Wiles
- A collection of makeover blogs so you can see examples at different grade levels
- Resources for low threshold, high ceiling, open middle, and productive struggle

CHAPTER 7: Mathematicians Ask Questions

In this chapter, we'll explore giving students opportunities to pose problems and ask questions, not just answer someone else's questions.

Discussion Questions

Page 137 In your math class's current state, how often do your students have opportunities to pose mathematical questions? How much time do you think you can manage for student questioning? Discuss or write.

Pages 150–152 Discuss the stock market independent study. What does it make you think about? Do you see opportunities for all your students to pursue their own interests in mathematics?

Pages 154–156 Discuss Debbie's choices around what to do with students' questions. How much freedom and structure did she provide? Her approach can sound scary. Especially if it does, make sure to talk it through with your colleagues.

Page 169 Now that you've read how it went, discuss the prior question again. What did you notice? What new thoughts are you thinking?

Page 169 Discuss or write about how Debbie handled vocabulary in the geometry exploration. What can you take away from her example?

Page 169 What do you make of the timing of students' questioning at the beginning, during, and at the end of this unit (rather than just at the beginning)? Discuss or write about *when* students can ask questions.

Page 169 Discuss the section on standards. Debbie never wrote an objective on the board, yet her students engaged in a rich exploration of the standards. In your teaching context, how might you give students opportunities to uncover the standards through inquiry?

Activities

Pages 139–141 **Play with 101questions**

Open up 101qs.com and play for a few minutes. Give yourself time to explore *before* you think about teaching it.

Pages 153–154 **Are Shapes Math?**

Allow yourself to be inspired by Debbie's kids. What questions do you have about shapes? Generate some. Coaches, perhaps you can find time for teachers to investigate their own mathematical questions.

CHAPTER 7: Mathematicians Ask Questions (continued)

Calls to Action

Pages 139–141 **Teach 101questions**

Try five different *101questions* with you students, as your schedule permits. How did it go? What changes did you notice as students gained experience? Share your findings at tjzager.com (Chapter 7).

Pages 141–145 **Notice and Wonder**

Read the blogs about *Notice and Wonder* (gathered at tjzager.com, Chapter 7), and then try a couple in your classroom. Pay attention to who participates. What do *you* notice? What do *you* wonder? Tell me about it at tjzager.com (Chapter 7).

Pages 145–147 **Problem Posing**

Consider Sheryl's investigation with the fish eggs. Is there a data set you could provide students so they could practice generating mathematical questions? Try it and see! Report your findings and read about other teachers' experiments at tjzager.com (Chapter 7).

Pages 147–150 **Riffing**

Choose a rich problem from an upcoming lesson and plan how you might give students the opportunity to springboard off their first solution. The questions "What new questions do you have?" or "What are you wondering about now?" might help. What new questions did students generate? Share them at tjzager.com (Chapter 7).

Additional Resources

At stenhouse.com/becomingmathteacher and at tjzager.com, you'll find a collection of supplemental resources that may come in handy for further thinking and discussion. I keep the links fresh, so the contents will change, but you will certainly find:

- 101questions
- Lots of information about *Notice and Wonder*, including several blogs from different grade levels and a short talk from Annie Fetter from the Math Forum
- References and articles about Van Hiele levels and resources for teaching geometry with questions at the heart, such as *Which One Doesn't Belong?* from Christopher Danielson

CHAPTER 8: Mathematicians Connect Ideas

Our students generally think of math as a series of discrete, disconnected topics, rather than as a landscape of interconnected ideas. In this chapter, I shared several teaching strategies to shift our focus to how ideas relate to one another.

Discussion Questions

Pages 177–184 Discuss the Jake's House problem vignette. How did Jen's use of students' representations help students learn mathematics and gain new perspectives?

Pages 194–200 How does Emily's story make you think about the role of connections in students' proficiency, or lack thereof?

Pages 201–203 Reflect on Jen's decision to give herself time to plan. Would you feel comfortable telling your students you needed this kind of think time? Can you give yourself permission to do so in the future? Discuss or write.

Page 207 Discuss the questions on page 207. Are you comfortable asking them? Practice discussing them with your colleagues until the language feels comfortable enough to use in class.

Pages 207–208 Discuss the end of the chapter with a trusted colleague, your notebook, or with fellow readers and me at tjzager.com.

Activities

Pages 203–204 **Staffing Your Faculty Lounge**

Do you have a community where you could talk about a student's work like I did? If not yet, please find me on Twitter at @tracyzager and check out https://exploremtbos.wordpress.com/ to learn how to join a thriving community. Christopher Danielson calls it the "faculty lounge of my dreams," and I agree. You get to staff it with exactly the people you want.

Calls to Action

Pages 174–176 **Families and Communities**

Try involving families and community members in math class, as suggested. How did you recognize their expertise and honor it with your students? Report back at tjzager.com (Chapter 8).

Page 176 **Social Justice Math**

Surf around rethinkingschools.org, radicalmath.org, and mathalicious.com. Find a lesson to try, or write your own. Teach it. How did your students react to engaging with social justice issues in math class? Reflect with one another, and talk about what you learned at tjzager.com (Chapter 8).

CHAPTER 8: Mathematicians Connect Ideas (continued)

Page 184 **Multiple Representations**

Choose a problem or set of problems and devote class time to analyzing, connecting, and comparing students' representations. If you can, please take pictures and post them to a blog or a Google Doc and share the link at tjzager.com (Chapter 8) so other teachers can see your student work. What did you learn? What did your students learn?

Pages 184–191 **"What does _____ have to do with _____?"**

Take some time to think about Deb's sentence frame, "What does _____ have to do with _____?" Choose two concepts from your grade band—concepts that are related in some way but students tend to think of as unrelated. What opportunities could you give students to connect these ideas? Write about what you decide and how it goes at tjzager.com (Chapter 8).

Pages 191–193 **Multiple Models**

Think about models you teach. Do your students currently see connections among them? What might you take from Becky's example? Try it, and share what you learned at tjzager.com (Chapter 8).

Page 207 **Appeals to Rules Versus Appeals to Mathematics**

Reflect on a time when you appealed to rules instead of the mathematics. What had a student tried? How would you handle the same situation now? Write about it at tjzager.com (Chapter 8) or blog about it and post the link.

Additional Resources

At stenhouse.com/becomingmathteacher and at tjzager.com, you'll find a collection of supplemental resources that may come in handy for further thinking and discussion. I keep the links fresh, so the contents will change, but you will certainly find:

- All the web resources from this chapter
- Skemp's paper and Wertheim's talk
- A recording of a webinar I gave about some of these ideas

CHAPTER 9: Mathematicians Use Intuition

This chapter has a fair amount of novel content for teachers. Intuition has historically played a minor role in teacher education and training, as well as in math class. My hope is its novelty leads to good surprises, new thinking, joy, and discovery. This chapter is intended to be a conversation opener, not the final word.

Discussion Questions

Pages 209–211 Discuss the opening passage about intuition. What caught your ear? Any surprises?

Page 214 Discuss Jen's approach to inserting exploration before the worksheet. She is able to stay on her pacing guide even with these insertions. What do you think about that idea?

Pages 216–219 Read the description of Alejandro's revision carefully. How might you model this thought process and teach it to your students?

Page 218 Study Figure 9.6. Discuss it with your colleagues, in your notebook, or on your blog. I'd love to hear your thoughts at tjzager.com (Chapter 9).

Pages 213–221 Reflect on Jen's use of manipulatives, and yours. Discuss how we might be more thoughtful about their use in classrooms.

Pages 221–224 Can you relate to what the teachers experienced with the roll of quarters problem? Do you see the same thing happen with your students?

Activities

Pages 238–242 **Developing Intuition in Students Who Need It Most**

Read Andrew Gael's comments with your full team—general and special education and paraprofessionals. Plan how you might use these comments to start a productive conversation.

Calls to Action

Page 221 **Inserting Intuition**

Try shuffling your lesson components, like Jen did, to make room for intuition building in an upcoming lesson. Don't expect it to go perfectly—Jen has a lot of experience with this technique! After you teach, reflect on what you noticed and thought. What might you try next time? Share with your colleagues, and, if you like, at tjzager.com (Chapter 9).

Pages 224–226 **Questioning**

Choose seven questions—one from each category—and write them somewhere you'll see them while you teach. Try using them in your teaching for a week. What did you notice? Share with your team and also at tjzager.com (Chapter 9).

CHAPTER 9: Mathematicians Use Intuition (continued)

Pages 230–239 **Estimation 180**

Teach a series of challenges from *Estimation 180*. How did it go? Share your reactions with your colleagues and at tjzager.com (Chapter 9).

Page 239 **Estimation 180 on the Number Line**

Once *Estimation 180* is a familiar routine, try it with Joe Schwartz's open number line. What did you learn? It was likely challenging for your students. Try it a few times and see how things evolve. Talk with one another and let me know how it goes at tjzager.com (Chapter 9).

Additional Resources

At stenhouse.com/becomingmathteacher and at tjzager.com, you'll find a collection of supplemental resources that may come in handy for further thinking and discussion. I keep the links fresh, so the contents will change, but you will certainly find:

- All the web resources from this chapter
- Blog posts from Joe Schwartz and Kristin Gray about *Estimation 180*
- Andrew Gael's blog so you can read more deeply about his work

CHAPTER 10: Mathematicians Reason

In this chapter, I dug into the different elements of mathematical reasoning and then got very practical. There are many classroom routines and techniques to try. Therefore, there's a lot to discuss as you make sense of the overall framework, and then there's a lot to experiment with in the classroom. I hope you'll lean on one another and reach out to me as you dabble with different ideas. Much of the content of this chapter has historically (and inappropriately) been saved for older students. We're in new territory. Let's work together.

Discussion Questions

Pages 244–245 What do you make of the argument that counterintuitive or paradoxical math is a motivator for proof? Discuss or write about this idea.

Pages 244–245 Discuss the list of counterintuitive concepts. Do you have any to add? Let me know at tjzager.com (Chapter 10).

Pages 272–274 Discuss this idea of leaving the door open to revision as students learn more mathematics. Karp, Bush, and Dougherty's (2014) "13 Rules That Expire" is a super article to read and discuss in conjunction with this section. It's linked at tjzager.com.

Page 278 Discuss or write about the final passage of this chapter, about intuition, mathematical reasoning, and sexism. What are you thinking right now? I'd love to hear your reflections at tjzager.com (Chapter 10).

Activities

Pages 247–252 **Elements of Mathematical Reasoning**

Working together with your colleagues, dig into this hypothetical story and analysis. Engage in raw thinking together. Talk it through. It may help you make sense of these terms and ideas.

Pages 257–261 **Visual Patterns**

Once you've read the section about visual patterns, go to mathtalks.net and work through the transcripts from at least five different pattern talks. You choose which five. Can you see the pattern the way the students saw it? After these five, choose a sixth pattern. This time, try to anticipate different student solutions before looking at the transcript. Any surprises? What did you learn?

Pages 265–269 **Game Night!**

After reading the section about games, peruse the resources at stenhouse.com/becomingmathteacher or tjzager.com. Play some games, watch some videos, and share what you learned with your colleagues.

Calls to Action

Pages 252–253 **Noticing Students Noticing Patterns**

For the next five math lessons you teach, keep a notebook with you. Jot down any patterns you hear students notice. Are they parenthetical comments, or do students seem to realize they are doing important mathematical work? Share your findings with your colleagues and tell me about them at tjzager.com (Chapter 10).

CHAPTER 10: Mathematicians Reason (continued)

Pages 253–258 **Choral Counting**

Read the section on *Choral Counting* carefully. Watch the suggested videos and explore *Choral Counting* on tedd.org. When you have a good sense of it, plan a count with your colleagues. Write a few ways you might record the count. When you're ready, teach the count, and then debrief with your colleagues. If you can snap a picture of your recording and link to it at tjzager.com (Chapter 10), I'd love to see it!

Page 262 **Questions to Encourage General Thinking**

Study this list of questions. Post them somewhere you'll see them. In the next week, choose a few and try them out with students. How did students respond? Talk with your colleagues about next steps, and feel free to report back at tjzager.com (Chapter 10).

Pages 263–264 **Open Number Sentences and True/False Number Sentences**

Choose one of these routines to try out. Plan it with your colleagues. Spend a long time choosing the numbers and the number sentence based on your mathematical goal for the lesson and what you've seen in your students' work. If you can teach it together, all the better. How did it go? Debrief together or in writing at tjzager.com (Chapter 10).

Pages 274–278 **Always, Sometimes, Never**

Plan and teach an *Always, Sometimes, Never* routine with your colleagues. As part of planning, read the blog posts about this routine that are collected at stenhouse.com/becomingmathteacher and tjzager.com (Chapter 10). You'll learn tips and techniques from your peers that way. After you've taught it, discuss the role of *ambiguity* in opening powerful conversations. How did the "sometimes" category encourage students to craft claims and consider conditions? Let me know at tjzager.com (Chapter 10).

Page 275 **Keywords**

Here's the firmest Call to Action I can suggest: please take any keywords posters down from your walls and remove any keyword lists from students' notebooks. Let's stop teaching math as if it's a code to crack, and instead teach students to make sense. *Always, Sometimes, Never* can help us do so.

Additional Resources

At stenhouse.com/becomingmathteacher and at tjzager.com, you'll find a collection of supplemental resources that may come in handy for further thinking and discussion. I keep the links fresh, so the contents will change, but you will certainly find:

- A lot of web resources for this chapter in particular, including many blogposts from colleagues and lots of videos of classrooms in action
- Further context about the role of proof. I've gathered highly readable, thought-provoking essays, blog posts, and articles
- Collections that will help you with the routines discussed in this chapter, such as banks of *Always, Sometimes, Never* statements, free games that encourage proof-like reasoning, and Fawn Nguyen's beautiful *Visual Patterns*

CHAPTER 11: Mathematicians Prove

Justification is a major focus of this chapter. What types of arguments do students tend to formulate? What types of arguments are credible to mathematicians? How can we teach students to draw on the argument structures mathematicians know and love so deeply?

Discussion Questions

Pages 279–280 Discuss or write about the different images for proof. Which resonates with you?

Page 282 Discuss or reflect on this quote: "When we argue about politics or chores or who started it, we're trying to get the other person to agree with us. When we argue for a mathematical claim, we're trying to establish the truth" (Balacheff 1991). What are the implications?

Pages 286–288 The concepts and vocabulary around inductive and deductive reasoning can be confusing. Make sure to take some time making sense of the examples together. You might want to try to create your own as well.

Pages 288–290 Discuss when your students tend to appeal to perception. Are there specific content areas where it comes up most? What might you do to disrupt their arguments the way Mary Beth Schmitt did by rotating the poster?

Pages 291–294 Discuss the section on the limits of cases. What are you thinking about? Are there things you're doing or saying, such as, "If it works for three examples, you can assume it's true," that encourage students to lean too hard on cases? How might you update your language?

Pages 294–295 Discuss the role of counterexamples and how you might encourage them more. Do you feel a connection between this section and the discussion of Jasmine's 38 × 12 conjecture in Chapter 8? Can you try to put their connection into words?

Pages 303–304 Discuss the architect's work versus the draftsmen's work from the Halmos quote. What thinking does it spark in you?

Pages 303–307 Open a conversation with your colleagues about the process of proving, as measured by verbs, compared to finished products. How might you assess students' mathematical reasoning? What do you think?

Pages 307–309 Write or talk about the "Proof and Equity" section. What changes in your thinking or practice would ensure that all students' ideas and thinking are valued?

CHAPTER 11: Mathematicians Prove (continued)

Activities

Pages 296–303 **Analyze Students' Justifications**

Read and discuss this section carefully. What are you thinking about how students use words, numbers, pictures, and gestures to prove? Once you've read it, read the Even and Odd Numbers blog posts from *Teaching Children Mathematics* linked at tjzager.com and stenhouse.com/becomingmathteacher. Discuss the two texts as paired texts.

Calls to Action

Page 283 **Sentence Frames for Proof**

Copy or otherwise post the sentence frames in the blue box. Begin introducing them to your students one at a time. Reinforce when students use them. After some time, reflect on whether you feel an impact on the tone of your discussions. Let me know at tjzager.com (Chapter 11).

Pages 283–284 **Set a Goal**

Spend some time as a team with this bulleted list. Through your discussion, it might be helpful for participants to identify a specific goal to work on in their practice. For example, some people might want to work on their poker faces. Others might want to focus on asking follow-up questions whether the answer is right or wrong. Choose a goal for yourself and try it in your teaching for two weeks. Hold one another accountable. Come back together and discuss how it went. Let me know at tjzager.com (Chapter 11).

Additional Resources

At stenhouse.com/becomingmathteacher and at tjzager.com, you'll find a collection of supplemental resources that may come in handy for further thinking and discussion. I keep the links fresh, so the contents will change, but you will certainly find:

- Links to related research articles from Harel and Sowder, Danny Bernard Martin, Stylianides, and more
- Optical illusions you can use to disrupt students' appeals to perception
- Mark Pettyjohn's lovely Math Celebration

CHAPTER 12: Mathematicians Work Together and Alone

The main focus of this chapter is to teach students to work in different ways, depending on their mathematical needs. Historically, we've focused on formats—individual work, partner work, group work—rather than the nature of the interactions or mathematical need. I hope I provided you with some compelling alternatives.

Discussion Questions

Pages 311–313 Discuss or write about this excerpt:

> ***If a major part of doing mathematics involves interacting with other mathematicians, then a major part of teaching students mathematics must be to teach students how, why, and whether to interact with one another mathematically. Students need to learn how to** ask for what they need from each other **and to** be what they need for each other. **In other words, we need to teach students how to be good colleagues, in math and in life.***

Page 314 I argued that we seek out collaborators when we genuinely need them. If we can solve a problem on our own, working in a group is more of a hindrance than a help. Discuss this assertion with your colleagues. What do you think? As you talk, it's fair game to draw on your experiences as a student as well as a teacher. Are you thinking about making any changes to your practice in light of this argument?

Page 314 Do your students think they have to be right in order to talk? Do you? What can you do to welcome partially formed thinking?

Page 320 Discuss or write about the paragraph at the top of the page, about socialization into the collaborative culture of mathematics. What do you make of it?

Pages 320–321 A key component of Fullilove and Treisman's "intervention" with underperforming students was to use "unusually difficult" problems. How does this approach compare to your school's current approach to intervention or remediation? If this question sparks deep thinking or changes to program, I'd absolutely love to hear about it at tjzager.com (Chapter 12). Their articles are linked on the website as well.

Pages 327–330 A common theme in this section is that exposing students to other ideas is not enough. Students need time and opportunities to try out strategies they pick up through cross-pollination. Reflect in writing or conversation about this assertion. Are there opportunities for cross-pollination that you're not converting? What changes might you make?

Pages 330–332 Talk about this idea of "credit" in the math classroom and mathematics. Have you seen students take offense if they're not given credit? How might you teach students to give credit graciously? I've linked a lovely Teaching Channel video of Laretha Todd talking about how her students give credit at the companion website. It's worth a look in this conversation.

Page 334 Discuss the way Shawn structured this class so students talked with people with whom they agreed and disagreed. How might you be inspired by this move?

CHAPTER 12: Mathematicians Work Together and Alone (continued)

Pages 339–345 From an equity point of view, many people make assumptions about Jen's (and Heidi's) students, including that they're not capable of engaging in cognitively demanding mathematics. One of my goals for this book was to provide honest examples of students of color engaging in conceptual, rich, mathematical explorations. I wanted to share substantial evidence that would challenge our implicit biases (my own included). This is a good place to pause and discuss this idea. What have you noticed about the capabilities and brilliance of young black and Latino/a mathematicians throughout this text?

Pages 345–347 Reflect on or discuss Jen's use of individual student critiques. What sort of teaching has to precede this assignment so it will be successful?

Pages 347–349 Look at this table together. What kind of teaching would it take for students to send and hear these messages? Reflect in writing or discuss with your colleagues. How can you make these cues explicit?

Activities

Page 317 **Planning for Collaboration**

Discuss the way Jen addresses collaboration at different points in the lesson. Take a look at the next lesson you're planning to teach. Where might be appropriate places to focus on the ways students are working together? How might you open those conversations? What will you look for?

Page 336 **Make Math Debatable**

Get together with your colleagues and tomorrow's lesson plans. Can you convert some of the questions into debatable questions? Practice together.

Page 339 ***Austin's Butterfly***

Watch this short film and discuss it with your colleagues. You might also consider showing it to your students. How does everyone react?

Pages 339–345 **Transcript Analysis**

Spend quite a bit of time reading this vignette from Jen's class. Make sure to analyze the mathematics in the student work and the transcripts. When you're ready, discuss this vignette with your colleagues. What do you think about the level of discourse and analysis from Jen's students? What is Jen doing to create the conditions for this caliber of conversation and critique?

CHAPTER 12: Mathematicians Work Together and Alone (continued)

Calls to Action

Pages 321–323 **Vertical, Nonpermanent Surfaces (VNPS)**

Take a look through the blogs about VNPS gathered at the companion website, and then figure out a way to get your students standing up and working on an erasable surface. Tell your colleagues and me about how you hacked your space challenges and what you noticed at tjzager.com (Chapter 12).

Pages 323–324 **Visibly Random Grouping (VRG)**

Liljedahl said that teachers saw a significant difference in the way their students worked with one another after three weeks of visibly random grouping. If you're ready for a little action research, can you commit to using VRG for three or four weeks and tracking your results? What do you notice? Share with your colleagues and me at tjzager.com (Chapter 12).

Pages 327–330 **Cross-Pollination**

Choose one of the suggested activities in this section and try it out with your class a few times. Share what you've learned with your colleagues in person and online at tjzager.com (Chapter 12).

Pages 335–338 **Sparking Debate**

Choose one of the activities or ideas from the description of Chris Luzniak's teaching and related resources. (There's quite a bit on the companion website and at tjzager.com, including videos of Chris in action.) Try it out with your colleagues and classes. Come back together and discuss. Share your thinking at tjzager.com.

Additional Resources

At stenhouse.com/becomingmathteacher and at tjzager.com, you'll find a collection of supplemental resources that may come in handy for further thinking and discussion. I keep the links fresh, so the contents will change, but you will certainly find:

- Several classroom videos and articles related to the content of this chapter
- A large collection of blog posts, especially around vertical, nonpermanent surfaces
- Chris Luzniak's videos

CHAPTER 13: "Favorable Conditions" for All Math Students

In this final, short chapter, I wanted to give you a chance to synthesize and extend all that has come before. I shared the work of one of my heroes, Dr. Clarence Stephens, and I gave you one last classroom example to reflect on. I hope readers will feel reflective at this point in their work together.

Discussion Questions

Page 352 Discuss the mission statement from Clarence Stephens's SUNY Potsdam math department. How does this statement compare with the mission of your institution?

Page 353 Have an honest discussion about the systems and supports you wish you had. Are there doable, concrete proposals you can make to your administration? Be specific about what you need to create favorable conditions for you, your colleagues, and your students.

Pages 353–356 Discuss the example from Ann Gaffney's classroom. Where are you now? What is standing out to you? What new questions do you have?

Page 356 This is a great spot to do some journaling, reflecting, and discussing. If you've made it this far as a group, how has the experience been? What have been the overarching themes of your work together? What ideas are you lingering over still? Going forward, what specific goals will you be working on? I'd be so grateful if you share your thoughts with me at tjzager.com (Chapter 13). I can't wait to hear what this learning has meant to you.

Additional Resources

At stenhouse.com/becomingmathteacher and at tjzager.com, you'll find further reading about Clarence Stephens and the Potsdam Miracle. Enjoy.

