Tania: So estimating, that's pretty close to 50 times 7, which is 350, right?

Janaya: What if we did 50 times 7 and then minus a 7, which is . . . 350 minus 7 . . . 343.

(Tania and Janaya wrote and thought. Maribel didn't write, but was clearly thinking.)

Maribel: We could do, like we could do 7 times 40 is 280. And 7 times 9 is 63. 280 plus 63 is . . .

Janaya: But we rounded to 50 and went back down.

Maribel: You did. It's easier for me to do it this way. So 280 plus 63 is 343. Is that what you found?

Tania: Yeah. We got the same thing two ways.

Janaya: Cool. That was a good way to check.

Maribel: So it equals 343. 343 *what*?

Tania: 343 . . .

(Long pause.)

Maribel: Let's reread the problem.

(They each reread the problem, and then thought quietly for about a minute.)

Janaya: Three hundred and forty-three cats. I think it's 343 cats.

Maribel: Oh my god. Each of the cats has 4 legs!

(Everybody laughed.)

Tania: Oh my god. That means 343 divided by 4 to figure out the legs.

Janaya: No, 343 times 4.

Maribel: Should we start over because we forgot we were thinking about legs?

Tania: Yes.

Here's the same group, several minutes later:

Maribel: So 7 big cats. No! 7 times 7 big cats. We have to do 7 times 7 again.

Tania: I'm just so confused. Too many sevens!

Janaya: When we get the answer of how many cats, we just need to multiply by 4 to find the number of legs.

Maribel: But humans have 2 legs!

Janaya: We're not counting the humans.

Maribel: We're not? Why not?

These bits of discourse are truly representative of the conversations happening around the room. Jen and I floated from group to group, jotting notes, listening in, and supporting students, and this is the caliber of conversation we heard from all groups. After twenty-five minutes of intense, active work, Jen stopped the class over howls of protest.

Jen: What do you think I'm going to say about how you did today?

Ethan: I think we did well because we were persevering on a hard problem.

Jen: Good word.

Eduardo: This is so hard, but I like it. I like getting confused!

Several students: Me, too!

Monique: I think we did very good because we were communicating with our partners and saying what we thought.

Jen: There was a lot of communication, and it wasn't about stuff we're not supposed to be talking about, right? In fact, sometimes it got kind of loud in here, but I didn't mind because I would say, like 98 percent of it was math talk, wouldn't you say?

Class: Yeah.

Joseph: You're going to say Joseph did awesome.

(Laughter.)

Jen: That's totally what I was going to say. Why am I going to say that?

Joseph: Because I didn't get frustrated when I didn't know the answer.

Jen: It's normal to get frustrated, but it's what you do when you get frustrated, right? You don't just give up!

Noemi: We were working really well with our partners because we were doing a really good job with each other when we disagreed. We didn't get up and leave. We stayed and talked it through.

Jen: You would sit there and defend yourself and say this is *why* I believe what I believe.

Jake: We've been working on this for like twenty minutes and we've gotten like five different answers, and we disagreed about all of them but we're still going.

Manuel: We've gotten like twelve different answers, and we still don't know which is right!

Class: Us, too!

Eric: We had like 9,576 and 9,814 and we got tons of different answers and my answer of 1,582, well I sort of think it's wrong but I sort of think it's right.

Maribel: Can we *please* keep working?

Jen: One last comment first.

Leon: I tried to hop in with them and try and defend my answer even though it was 37,530 and they said I was wrong, but I tried to defend my answer.

Max: He did defend his answer.

Manuel: You did a good job defending your answer.

Jen: I saw that you guys were getting a little frustrated over here, and I think what happened was—if you are still trying to solve something and you're not sure about what you think yet and someone says, "Here's what I think!" you might not be *ready* to hear yet because you're still trying to think about what *you* think. I get like that. If someone tries to talk to me about something I haven't thought about yet, I get like, "No! Don't talk to me!" (Jen covered her ears as a gesture. Lots of kids nodded at this.)

Jen is teaching students that empathy, communication, reading social cues, disagreement, building shared norms, and collaboration are part of doing mathematics. They are not add-ons or lessons that we only do during morning meeting or as part of an antibullying curriculum. They are not for September only, and they are not indulgences. Socialization into the collaborative culture of mathematics is, in fact, part of learning mathematics. It's what mathematicians do and is as important content as fractions.

The Educational Payoff, Especially for Marginalized Students

In the 1970s and 1980s, Robert Fullilove and Uri Treisman conducted a seminal study of college students at University of California, Berkeley, that showed another reason why we need to teach students to work together mathematically. The researchers were seeking explanations for why, in calculus class at Berkeley, "African American students were disproportionately represented among the ranks of the weak students" and "Chinese Americans were disproportionately represented among the strong students" (Fullilove and Treisman 1990, 465). In the decade before the study, 60 percent of black students who took calculus at Berkeley received grades of D or F. Why?

Their study quickly revealed that the researchers' stereotypes and assumptions about black students at Berkeley were unfounded. Black students were highly motivated, had been adequately prepared for college math, and had strong family support for higher education. Income levels didn't correlate either. So what was really causing the gap?

Fullilove and Treisman studied all the variables they could generate and found the major difference lay in how the two groups studied: the African American students diligently studied as many hours as the professor suggested, but they *studied alone*. The Chinese American students studied independently as well for about the same amount of time but also spent several, additional hours each week *studying together*. In other words, they studied more, and they studied more together:

> In the evenings [the Chinese-American students] would get together. They might make a meal together and then sit and eat or go over the homework assignment. They would check each others' answers and each others' English . . . They would edit one another's solutions. A cousin or older brother would come in and test them. They would regularly work problems from old exams . . . They had constructed something like a truly academic fraternity. (Treisman 1992, 366)

In response to these findings, Fullilove and Treisman formed honors mathematics workshops to teach students how to work together—to make studying mathematics social. They organized students into groups that worked together, twice per week, two hours per session, on "carefully constructed, unusually difficult problems" (Fullilove and Treisman 1990, 468) and encouraged students to discuss their thinking. Students in the math workshop spent about half the time working together, and the other half working independently.

The results were stunning. Over twelve years of study and hundreds of participants,