

	EXAMPLE REQUESTS	UNSPOKEN MESSAGES	HOW TO HEAR THE REQUEST
THINKING PARTNERSHIPS	<p>“Would you help me think something through?”</p> <p>“Can I bounce an idea off you?”</p> <p>“Can I think out loud with you for a second?”</p> <p>“Hey, look what I noticed. What do you think about it?”</p> <p>“Can we work on number 5 together? It would really help me to talk it through with somebody.”</p>	<p>I need to talk through this problem while I am trying to figure it out. I want to do more than solve it; I want to use this problem to learn. I'm looking for a sympathetic listener and a supportive partner who will join with me. We will both say things that are right and things that are wrong, and that's part of making sense. We'll create a safe space to share raw, partial, and unedited thinking with each other. By the end, I hope we'll both learn a lot.</p>	<p>This person needs someone to work alongside him in a collaborative way. We'll probably go back and forth between working independently and sharing ideas together. I need to listen carefully, be honest, and make it safe for us both to make mistakes and revise our thinking as we go. I should be encouraging and join with him. If he doesn't understand something, I'll ask him questions that will help him figure it out for himself. He'll do the same for me. We'll both make sure we're both learning, and we'll take the time we need to understand.</p>
CROSS-POLLINATION	<p>“I'm stuck. It would help me to hear what some other people have tried.”</p> <p>“I'm thinking about this problem in one way, but it would help me to hear some other perspectives.”</p> <p>“Can we walk around and look at other people's work? I need some new ideas.”</p> <p>“What have you been thinking about in your group?”</p>	<p>I feel like I've run through my ideas and I'm not getting where I want to go. I need some input from other people. Maybe somebody else's perspective will help me figure out where to go next. I will give credit to other people for their ideas, of course, and share my ideas with them, too. Their reactions to what I've done might help them or me.</p>	<p>This person needs to hear some different ideas to spark her thinking. She's not copying or stealing; she's looking for inspiration. That's a good thing! I will share my ideas in the hopes they help her, and in the hopes that people's reactions to what I've done will help me. I'll listen to everyone carefully because I've learned that my peers are amazing sources of mathematical ideas.</p>
MATH DISPUTES AND DEBATES	<p>“I'm not sure what's true here. If we argue opposite positions, we might be able to figure it out together.”</p> <p>“Would you debate this with me?”</p> <p>“I need someone to argue this through with me.”</p> <p>“OK. I'm going to act like I accept this claim. Can I argue it with you?”</p>	<p>The process of constructing and critiquing arguments will help me clarify my thinking about this concept. I need a partner who will listen carefully and engage in productive debate with me. We can use the structure of debate to help us both figure out what's true. Even though we're positioning ourselves on opposite sides of an argument, we are working together to determine the truth, understand mathematics, and articulate our thinking.</p>	<p>This person needs to debate in order to figure out the math. We'll disagree or agree with each other's ideas, remembering that our debate isn't personal. Through the process of debating, I'm sure my arguments will get sharper, and I'll help her make her arguments better also. I hope our debate will be lively, and we'll make sure it is respectful.</p>

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PEER FEEDBACK	<p>"I need a critical friend to look at my work."</p> <p>"Would you read my argument and see if you can find any weaknesses to attack?"</p> <p>"I made this as clear as I can, but I'm not sure it's clear to a new reader. Would you read it for me and give me feedback?"</p> <p>"Could you critique this for me?"</p>	<p>We all need outside eyes to make our work better. I need someone to critique what I've done and help me improve it. When someone criticizes my argument, they are not criticizing me. They are helping me strengthen my mathematical work, and I won't take it personally. I want to hear what they thought I did well so I can keep doing it, and identify and improve the parts that need work.</p>	<p>This person needs me to be skeptical and critical and look for flaws in his argument so he can make it better. In my own work, I value meaningful feedback I can act on, rather than empty praise, so I'll try to give him thorough, constructive comments. I'll make sure to give some positive feedback about the strengths, and then useful suggestions about areas that could be improved.</p>

When students have internalized these different interactions and essential habits of mathematical practice, they flow from one type of interaction to another, organically, as their needs vary and evolve. They act and think like the young mathematicians they are.