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MATHEMATICIANS TAKE RISKS

Mr. Duncan (pseudonym) was my eighth-grade algebra teacher. I would describe him as a methodical but uninspired math teacher, and we marched through the book learning procedures and mnemonics. I vividly remember the day he taught us the "rule" that any number raised to the zero power equals one, written $a^0 = 1$. I had been fairly comfortable with exponents up until then, thinking they were a nifty notation for writing big numbers. I had been taught to think of 2^4 as shorthand for two multiplied by itself four times: $2^4 = 2 \times 2 \times 2 \times 2 = 16$. Raising two to the zero power, however, was a new idea, and I was having trouble imagining it.

I sat in the back row of the class, wondering whether 2^0 should equal zero, because it was two multiplied by itself zero times. Or maybe it should equal two, because it was a two nobody seemed to do anything to? No matter how I looked at it, though, I couldn't figure out where a one would come from. I raised my hand for help. My conversation with Mr. Duncan went something like this:

"I don't understand where the one comes from."

"It comes from this rule I just taught you, $a^0 = 1$."

"But why is that the rule? It doesn't make sense to me."

"Because that's the rule."

"I could kinda see the answer being zero, or a, but how do we get a one? There's no one in the problem. It feels like you pulled it out of thin air."

(Irritated.) "It comes from the rule."

"But how can one be the answer for any value of a? How can $5^0 = 1$ and $20^0 = 1$ and $1000^0 = 1$? I don't see how all those problems can be equal, and why are they all equal to one? What does one have to do with anything?"

(He sounded truly exasperated now.) "You're right. Any number raised to the zero power is one. That's the rule you need to memorize."

"But . . ."

At this point, my classmates joined my teacher. The popular kid I had a crush on said, "Give it up, Tracy! Who cares?" A chorus followed him, and everyone started laughing, including Mr. Duncan. This was eighth grade. I gave it up, pretending to laugh on the outside.

On the inside, I was angry. I was a good student trying to understand. At the time, I felt like Mr. Duncan tried to shut me down, and it worked: I stopped raising my hand in class. In hindsight and as a teacher, though, I think I understand Mr. Duncan's behavior better. My guess is he didn't know the answer to my question. He had been taught to memorize exponent rules, just like he was now teaching us. Rather than seeing my curiosity as an opportunity to go deeper into the mathematics and build real understanding—mine and his—he likely perceived my questioning as a public challenge to his expertise and authority. He was standing at the front of the class, trying to save face. Shutting me down wasn't his goal, I'm sure (or at least I hope!), even though that was the effect of his actions.

When I listen to teachers talk about their experiences as math students, a story similar to mine often emerges: they were (1) trying to understand a math concept, so (2) they asked their teachers (or parents) a lot of questions until (3) the teachers (or parents) ended the discussion with a thud:

> "Because that's the rule."
>
> "You just need to memorize it."
>
> "I know it's confusing, but you need to learn it."
>
> "Stop worrying about why. This is how you do it."
>
> "It's in the book; that's why."

These sorts of responses certainly lead to negative feelings about math. Students feel confused, baffled, incompetent, frustrated. Terrible! In addition to all that, though, these sorts of moments contribute to a pervasive, damaging myth about mathematics as a discipline, which is that *obedience* is ultra-important. We're just supposed to quiet down and abide by the rules. If we follow the teacher's procedures, lining everything up and doing the steps in order, we'll get the correct answer, right?

The thing is, obedience is unmathematical. Mathematicians, as a group, are a ragtag bunch of rebels. They push boundaries, ask questions, take risks, test conjectures, try new things, and are relentlessly passionate about pursuing meaning. As mathematician Paul Lockhart (2009, 31) put it:

> Math is *not* about following directions, it's about making *new* directions.

Mathematicians are most excited by problems that have no known answers, steps, or "right ways to do it." These problems are called *unsolved*, or *open*, problems, and they drive the field. By seeking to understand what is not yet understood, mathematicians discover and invent new techniques, models, tools, ideas, connections, and structures. Their goal is to create new mathematics, not to replicate, practice, or regurgitate existing methods, and they certainly have no answer key!

For mathematicians to tackle open problems, they must take risks. They need to be bold enough to try novel approaches, including far-fetched ones that have a high likelihood of failure, because new thinking is what's needed to solve a problem that's been stumping everybody else. It takes a certain amount of gumption to say, "Newton died unable to solve this problem, but I think I'll give it a go."

Mathematicians head into uncharted territory without maps, intentionally walking away from the familiar and the known, hoping to forge new trails somewhere interesting. In short, mathematicians have moxie. Consider the terms of praise they reserve for the most important solutions to the most captivating problems: *innovative, breakthrough, surprising, daring, revolutionary, creative, beautiful, subversive, elegant, imaginative!* A far cry from the forced compliance of "Because that's the rule you need to memorize," aren't they?

Perhaps you're thinking that pluck, innovation, and risk taking apply only at the frontiers of mathematics and only to research mathematicians? After all, we're not inventing any new math in school, are we? Don't we just teach "the basics"?

The answer is no, thank goodness!

Risk Taking Among Young Mathematicians

Imagine a young girl walking on a sidewalk, counting her steps on her way home from first grade. Once she gets to fifty, she decides to switch and count down instead of up. As she approaches zero, her stride slows, but she still has more steps to take. Her jaw drops as she wonders whether there are numbers on the other side of zero. She can almost see them stretching out in front of her. She may not know their conventional name, but she has just had the same epiphany the great Indian mathematician Brahmagupta had in the seventh century when he invented negative numbers. Don't let our society's acceptance of these radical ideas take anything away from the breakthrough that girl just made! She is thinking like a mathematician, daring to wonder about limits and possibilities, and her ideas are delightful.

When students generate real mathematical understanding, they are creating math anew. Each time a child has an aha! moment about place value, makes a connection between fractions and division, or discovers rotational symmetry in a flower, that child is reinventing mathematics. Whether or not other mathematicians have had the same idea before is *completely irrelevant to that student*. The inverse relationship between addition and subtraction may be settled mathematics to us, but it's an open problem for every single young mathematician we teach.

For our young mathematicians to tackle open problems, they must tinker with mathematics, make leaps, ask questions, share their ideas, and handle frequent failures in math class, and we must make them feel safe and encouraged to do so. Mathematician James Tanton (2012) said, "Math is being able to engage in joyful intellectual play—and being willing to flail (even fail!)." As educators, we must teach students how to engage in joyful intellectual play, flail, and even fail as they muck around with math, because that is how they will construct new mathematical understanding for themselves.

I often hear teachers commiserate that their students won't take risks, won't try anything new, won't start a problem if they can't see a short, clear path from the start to the finish. When conversation shifts toward what we do to encourage risk taking in the math classroom, however, I hear a lot of messaging strategies and not much else:

> "I *tell* my students that the only way to learn something new is to try."
>
> "I *tell* my students that I make mistakes all the time."
>
> "I *tell* my students that taking risks is part of learning."

These messages are all well and good, but it's going to take a lot more than spoken messages and inspiring posters to teach students how to take mathematical risks, especially if they've already been socialized into a culture of passive obedience in mathematics. It's going to take *teaching. Actions. Instructional strategies.* If we want students to line up quickly and quietly, we must teach them how. If we want students to take quality notes, we must teach them how. And if we want students to take risks in math class, we must teach them how!

In this chapter, we'll learn from three teachers who have developed specific instructional strategies to teach them how:

- We'll visit Heidi Fessenden and see how she values and highlights risk taking publicly as part of teaching mathematics.
- We will analyze student work samples from Cindy Gano to see how she uses brief, specific, written feedback to encourage risk taking.
- We'll look at how Shawn Towle establishes a classroom culture that is *both* rigorous and safe, and see how he leverages his leadership to support students while they take risks.